The last decade witnesses rapid progress in studies of scattering amplitudes in gauge theories. The progress has been made possible in hand with a deeper understanding of string theory and two-dimensional conformal field theory (CFT). The first indication of a relation between four-dimensional gluon amplitudes and two-dimensional CFT was pointed out by Nair in 1988, that is, at tree level the so-called maximally helicity violating (MHV) amplitudes can be interpreted as the correlators of the Wess-Zumino-Witten (WZW) model suitably defined in supertwistor space [1]. In other words, as long as the MHV amplitudes are involved one can describe the four-dimensional gluon amplitudes in terms of two-dimensional CFT. This relation was further extended to the non-MHV amplitudes by Witten in 2004, where a weak-weak duality between $\mathcal{N} = 4$ super Yang-Mills theory and topological string theory was presented [2].

Soon after Witten's extension, a flood of new results on various aspects of the amplitudes came out. In particular, the Britto-Cachazo-Feng-Witten (BCFW) recursion relations play an essential role in analytic and algorithmic developments. Another important result is the dual superconformal invariance of the amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory. This unveils a connection between the amplitudes and CFT at the level of symmetry. Other results, such as Yangian invariance, correspondence between amplitudes and Wilson loop operators, have also reported in these periods. These new results suggest that there should be some alternative mathematical formulation of scattering amplitudes, which is qualitatively different from the traditional Fyenman-diagram approach.

In 2008 Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot and Trnka showed that such a formulation is indeed possible. They found that for the calculations of loop amplitudes in accord with these new results, in particular, the BCFW recursion relations and the Yangian invariance, the most powerful and mathematically sophisticated formulation of the scattering amplitudes can be given in a form of integrals over Grassmannian spaces [3]. Or more precisely, they proposed a reformulation of the amplitudes in terms of contour integrals over the Grassmannian spaces $Gr(k, n)$.

The purpose of the book under review is to present an accessible introduction to this intriguing new endeavour of research. The book is mostly based on the originally-prepared arXiv reference [4]. This was written in the midst of what someone called the mini-revolution of scattering amplitudes and, since then, it has become one of the guiding forces for attentive researches.

The book covers basic elements in the recent developments of scattering amplitudes, which include the spinor-helicity formulation, the supertwistor spaces, the BCFW recursion relations, the dual superconformal symmetry, and the Yangian symmetry. For recent reviews on these elements, the reader may also refer to [5, 6, 7]. The book also presents a pedagogic description of combinatoric properties of the amplitudes and Grassmannian representations of on-shell diagrams. It contains a variety of figures to facilitate the understanding of analytic and geometric properties in the Grassmannian formulation. In latter chapters, it also deals with applications of the Grassmannian formulation to advanced topics such as Yang-Baxter relations, ABJM theories, and cluster algebras.

The focus of the book is the introduction to the Grassmannian formulation of the scattering amplitudes. The amplitudes of interests here are planar and on-shell scattering processes of gluon. Massive particles have not yet come into the formulation in this book. In this sense, the book does not provide a direct clue for phenomenologically oriented readers; it rather concentrates on mathematical aspects of the amplitudes and would be useful for both mathematicians and theoretical physicists in general. The Grassmannian formulation is also proven to be powerful to a systematic description of gravity amplitudes but this topic is not covered in the book; interested readers may easily access to relevant references at the online arXiv.

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