Recent developments in the computation of scattering amplitudes in gauge theories at loop level show an intriguing relation to (ordinary) polylogarithms and multiple polylogarithms. Mathematically, it is known that such polylogarithms can be expressed in terms of iterated integrals [1, 2], which enable us to relate various polylogarithms to one another. These relations are, in turn, potentially useful to simplify loop calculations of gluon amplitudes.

This article is prepared by one of the experts on this particular subject where we find fruitful interplay between modern mathematics and physics. The author introduces algebraic tools for the analysis of multiple polylogarithms, such as 'symbols' and 'extended Hopf algebra', in part reviewing the result of [3]. The author then applies the results to a specific one-loop calculation, the so-called one-mass one-loop box function, which arises in typical one-loop Feynman integrals. For two-loop calculations and other examples, the author also provides recent references.

This article pays attention to recent progress in number theory and its applicability to amplitude calculations in quantum field theory. On this regard, one may be interested in a very recent study on possible correspondence between Yang-Mills theory and the ABC conjecture [4]. As far as the reviewer notices, there does not yet exist a research on scattering amplitudes in connection with modern number theoretic concepts, such as class field theory, Iwasawa theory, or correspondence between primes and knots [5].

References

[1] A. B. Goncharov, Duke Math. J. **128**, 209 (2005) doi:10.1215/S0012-7094-04-12822-2 [arXiv:math/0208144 [math.AG]].

[2] T. Kohno, *Conformal Field Theory and Topology*, Translations of Mathematical Monographs, Volume 210, American Mathematical Society (2002).

[3] F. Brown, arXiv:1102.1310 [math.NT].

[4] Y. H. He, Z. Hu, M. Probst and J. Read, arXiv:1602.01780 [hep-th].

[5] M. Morishita, "Analogies between Knots and Primes, 3-Manifolds and Number Rings," arXiv:0904.3399 [math.GT]; *Knots and Primes* (in Japanese), Springer-Japan (2009).