In this article the authors determine counterterms of four-dimensional $\mathcal{N} = 1$ supersymmetric field theories on curved space by considering a rigid limit of the so-called new minimal supergravity [1, 2] in Euclidian signature. The rigid limit of off-shell supergravity theories, which can be taken by sending the Planck mass to infinity, has literally no dynamical degrees of freedom and is known to define relevant supersymmetric field theories on curved manifolds; see [3] for recent study on this issue. The supersymmetric counterterms are supergravity invariant. Demanding further the invariance under diffeomorphism and R-symmetry transformations and utilizing the previously known results of new minimal supergravity, the authors then obtain the counterterms of the 4-dim supersymmetric field theories on curved manifolds in forms of both dimensionless and dimensionful integrals.

The resultant counterterms are evaluated on supersymmetric background. On a bosonic background, where we can ignore fermionic ingredients, the counterterms can be simplified. One of the main results of this article is that all dimensionless (also called marginal/finite) counterterms vanish when the bosonic background admits two supercharges of opposite R-charge. This suggests that in this case one can construct a 4-dim supersymmetric field theory uniquely as a quantum theory, that is, one can define a theory such that it is independent of the choice of renormalization schemes.

Interestingly, partition functions on Hopf surfaces given in [4] fit into the above mentioned case. Supersymmetric Casimir energy computed in [4] is therefore expected to be a physically meaningful quantity. In fact, as presented in the well-written first section, one of the main purposes of this article is to find physically meaningful observables out of 4-dim supersymmetric field theories. The above Casimir energy provides a concrete answer to such a primary purpose. Recently, this Casimir energy is further scrutinized and justified in comparison to ordinary (non-supersymmetric) Casimir energy; see [5] for details.

References

[1] M. F. Sohnius and P. C. West, Phys. Lett. B **105**, 353 (1981).

[2] M. Sohnius and P. C. West, Nucl. Phys. B **198**, 493 (1982).

[3] G. Festuccia and N. Seiberg, JHEP **1106**, 114 (2011) [arXiv:1105.0689 [hep-th]].

[4] B. Assel, D. Cassani and D. Martelli, JHEP **1408**, 123 (2014) [arXiv:1405.5144 [hep-th]].

[5] J. Lorenzen and D. Martelli, arXiv:1412.7463 [hep-th].