The moduli space of four-dimensional $\mathcal{N} = 2$ supersymetric theories in view of string compactification is known to be factorized by the vector-multiplet and the hypermultiplet moduli spaces (see [1] for the background of this subject). The vector multiplet moduli spaces had been investigated thoroughly in 1990s, leading to deep connections with algebraic geometry and string theory, and, in particular, to the discovery of the so-called mirror symmetry of CalabiYau manifolds.

The understanding of the hypermultiplet moduli space, on the contrary, has been far less completed. This is largely due to the fact that the hypermultiplet moduli space is a quaternion-Kähler manifold, required by the $\mathcal{N} = 2$ extended supersymmetry. The quaternion-Kähler geometry has a rather cumbersome property than the (special) Kähler geometry of the vector multiplet counterpart. Recently, however, computational techniques to deal with this problem has been developed by use of the complex structure of the twistor space (see [2] for a review).

Motivated by these developments, in the article under review, the authors further make use of the following results:

1. the duality between heterotic and type-II string theories, more precisely, the duality between $E_8 \times E_8$ heterotic string theory compactified on a K3 surface times T^2 and type-II string theory compactified on a suitable Calabi-Yau threefold; and

2. the recently discovered map between hetrotic and type-II moduli in the classical limit [3].

The authors then show a new twistorial parametrization of the hypermultiplet moduli space in the classical limit. The parametrization is new in a sense that it is of direct relevance to the symmetries of the heterotic string theory. Quantum corrections to the hypermultiplet scalars are also considered in the article. Readers interested in these developments are also recommended to refer to a recent review [4].

References

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