In this article the authors further study a novel formulation [1, 2] for the calculations of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory in terms of integrals over the Grassmannian $G(k, n)$, *i.e.*, $(k \times n)$ matrices modulo a $GL(k)$ action on the right. One of the advantages in this formulation is that it makes the Yangian invariance of the amplitudes manifest. This symmetry is recently revealed in multi-particle analyses of the scattering amplitudes; see, for example, [3, 4].

The physical configuration space of an n-gluon system within the spinor-helicity formalism is given by $\mathcal{C} = \mathbf{C}^n/\mathcal{S}_n$ where $\mathbf{C}$ denotes the complex (projective) space on which a spinor momentum of each gluon is defined and $\mathcal{S}_n$ denotes the rank-n symmetric group. The fundamental homotopy group of $\mathcal{C}$ is known to be the braid group, $\Pi_1(\mathcal{C}) = B_n$. The Yangian symmetry is intimately related to the braid group. The multi-particle analysis therefore provides an interesting new perspective to gauge theories in general, compared to the ordinary single-particle, or solitonic, analysis of the gauge theory where the fundamental group of the corresponding physical configuration space is usually given by $\mathbf{Z}$.

In this article an intensive study along this basic idea is explored, focusing on a Grassmannian viewpoint. From the previous studies, it has been known that a certain sum over the residues of the Grassmannian integrals can lead to the physical amplitudes. In this article the authors show that such a sum can be understood in a unified fashion in terms of an algebraic variety. Further, motivated by this unified picture, the authors propose to interpret the integrals as "particles" in the Grassmannian. This proposal seems to be radical and somewhat farfetched, however, as shown in the article, it does agree with the connected prescription for tree amplitudes in Witten's twistor string theory [5, 6]. The results are also expected to be useful for the calculations of loop amplitudes.

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