Recently there is much attention to the so-called Kawai-Lewellen-Tye (KLT) relation between tree-level amplitudes of closed and open strings [1]. Apart from calculatory developments in string amplitudes, this is mainly motivated by recent progress in the understanding of gauge theory amplitudes and its relation to the gravity theory counterparts in the context of twistor string theories. Regarding the KLT relation, it is shown for the strings that are moving on a sphere S^2. In this article, an argument is made that, if we change the shape of S^2 to either a disk D_2 or a real projective plane $\mathbf{RP}_2$, the KLT relation no longer holds. The mismatch seems to arise from particular topological properties, *i.e.*, a boundary effect in the case of D_2 and a crosscap effect in the case of $\mathbf{RP}_2$. The authors find that, for strings defined on D_2 or $\mathbf{RP}_2$, tree-level amplitudes of closed and open strings can be linear to each other, while in the ordinary S^2 case closed string amplitudes factorize into two sectors of open string amplitudes that correspond to right-moving and left-moving open strings, respectively. The result is suggestive that we can more generally express closed string (or gravity theory) amplitudes as summations of open string (or gauge theory) amplitudes, with an appropriate choice of combinatoric phase factors.

References

[1] H. Kawai, D. C. Lewellen and S. H. H. Tye, Nucl. Phys. B **269**, 1 (1986).