

This article deals with a novel proposal on the construction of quantum field theories in noncommutative geometry [1, 2] where the underlying algebra is encoded by the so-called holonomy-diffeomorphisms algebra [2]. This proposal has been developed by the authors for many years, initially as a mathematical framework for quantum gravity in some connection to gauge theories. Incorporation of spinor fields is also studied in [3].

In this article the authors further consider incorporation of Dirac operators. One of the main results is that self-dual and anti-self-dual sectors of gauge theories can be obtained from a square of a unitary transformed Dirac operator where the unitary transformations are described by Chern-Simons terms. The results would deepen the understanding of the holonomy-diffeomorphisms algebra. Interested readers may also refer to [4] for recent developments.

## References

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