Conformal field theories on two-dimensional lattices have been studied for decades; see, *e.g.*, [1] for early-stage developments. Recently, the concept of additive codes in the lattice construction of 2-dim conformal field theories has been further investigated in [2, 3]. Given that 2-dim lattices naturally describe tori or topological defects, one can expect that the additive codes somehow relate to invertible global symmetries generated by the topological defects. In this article, using 3-dim abelian Chern-Simons theories (which are holographically dual to 2-dim conformal field theories on the lattice), the authors clarify the relation between the additive codes and the global symmetries. Namely, the additive codes arise as subgroups of the group $G = \Lambda^*/\Lambda$ of invertible global symmetries where $\Lambda \in \mathbb{R}^{n,\tilde{n}}$ denotes a lattice, with Λ^* being its dual. To be more technical, the authors show that even codes correspond to non-anomalous subgroups of G and even *self-dual* codes correspond to *maximal* non-anomalous subgroups of G by calculating wave functions of 3-dim abelian Chern-Simons theories and partition functions of 2-dim conformal field theories on the lattice. The authors also consider partition functions with a sum over all possible topologies and argue that these functions describe gravitational theories.

References

[1] L. Dolan, P. Goddard and P. Montague, Commun. Math. Phys. **179**, 61-120 (1996) doi:10.1007/BF02103716 [arXiv:hep-th/9410029 [hep-th]].

[2] A. Dymarsky and A. Shapere, JHEP **21**, 160 (2020) doi:10.1007/JHEP03(2021)160 [arXiv:2009.01244 [hep-th]].

[3] N. Angelinos, D. Chakraborty and A. Dymarsky, JHEP **11**, 118 (2022) doi:10.1007/JHEP11(2022)118 [arXiv:2206.14825 [hep-th]].