Recently, a partition function of $U(1)$ Chern-Simons theory is considered in relation to Deligne-Beilinson cohomology [1]. This result is generalized to $U(1)^n$ Chern-Simons theory, leading to the construction of a $U(1)^n$ partition function in terms of the Deligne-Beilinson cohomology [2]. In this article the authors study some properties of the $U(1)^n$ partition function. For example, applying a reciprocity formula obtained in [3], the authors show that the partition function is a topological invariant (named a Reshetikhin-Turaev invariant) of a closed oriented 3-manifold. The authors also indicate reciprocity among the $U(1)$ partition functions by exchanging the roles of the so-called linking matrix and the coupling matrix. This relation is analogous to the reciprocity for Gauss sums on finite abelian groups [4] and would be useful for applications of topological quantum field theory to topics in elementary number theory.

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