In this article the authors study the 2-dimensional $\mathbb{CP}^{N-1}$ sigma model on $S^1 \times \mathbb{R}^1$ where the S^1 is parametrized by a position-dependent θ angle so as to investigate the θ dependence of this particular bosonic model. The authors consider the small S^1 compactification with nonzero winding θ and a suitable symmetry twisted boundary condition in the model. The system then obtains N classical vacua connected by fractional instantons. For details on this model, one may also refer to relevant studies, see, *e.g.*, [1, 2, 3, 4].

One of the main results in this article is that for nonzero winding θ, due to anomaly constraints, the quantum vacua of the system should have N-fold degeneracy; note that the condition for θ being a nonzero winding number is given in eq. (1.1). In other words, the fractional-instanton amplitudes of the system vanishes for nonzero winding number θ at quantum level. The authors show this result by demonstrating that an integration over the zero modes annihilates the fractional-instanton amplitudes, adding some interpretation in connection to an effective theory of the $SU(N)$ spin chain system. The authors also indicate that a similar phenomenon occurs in a basic quantum mechanical system with the Berry phase.

References

[1] G. V. Dunne and M. Unsal, JHEP **11**, 170 (2012) doi:10.1007/JHEP11(2012)170 [arXiv:1210.2423 [hep-th]].

[2] Z. Komargodski, A. Sharon, R. Thorngren and X. Zhou, SciPost Phys. **6**, no.1, 003 (2019) doi:10.21468/SciPostPhys.6.1.003 [arXiv:1705.04786 [hep-th]].

[3] Y. Tanizaki, T. Misumi and N. Sakai, JHEP **12**, 056 (2017) doi:10.1007/JHEP12(2017)056 [arXiv:1710.08923 [hep-th]].

[4] C. Córdova, D. S. Freed, H. T. Lam and N. Seiberg, SciPost Phys. **8**, no.1, 001 (2020) doi:10.21468/SciPostPhys.8.1.001 [arXiv:1905.09315 [hep-th]].