In this paper the authors study effects of curvature couplings in 2-dimensional noncommutative Φ^4 theory, or the so-called Grosse-Wulkenhaar model [1], utilizing both analytic and numerical approaches. Here the scalar filed Φ is given by an $N \times N$ hermitian matrix. To carry out an analytic approach, the authors first neglect the kinetic term and use large N approximation so that the curvature field R is represented by an N-dimensional diagonal matrix. The curvature term of interest is given by $S_R = N\mathrm{Tr}\,(-g_r R\Phi^2)$ where g_r denotes the curvature coupling.

After the diagonalization of Φ the curvature term gives rise to off-diagonal elements. To deal with this problem, the authors take advantage of the so-called Harish-Chandra-Itzykson-Zuber integral, see *e.g.* [2], to describe the action of interest in terms of multitraces; the action (3.20), one of the main results of this paper. This enables the authors to execute analytic and numerical analyses of phase diagrams of the matrix model (3.20) with fixed g_r and (large) N.

Regarding the numerical approach, the authors make use of a recently developed technique in Monte Carlo simulation [3] and show that the results are in accord with the previous numerical studies in a closely related matrix model [4].

References

[1] H. Grosse and R. Wulkenhaar, "Renormalization of phi**4 theory on noncommutative R**2 in the matrix base," JHEP **12**, 019 (2003) doi:10.1088/1126-6708/2003/12/019 [arXiv:hep-th/0307017 [hep-th]].

[2] B. Eynard, T. Kimura and S. Ribault, "Random matrices," [arXiv:1510.04430 [math-ph]].

[3] S. Kováčik and J. Tekel, "Eigenvalue-flipping algorithm for matrix Monte Carlo," JHEP **04**, 149 (2022) doi:10.1007/JHEP04(2022)149 [arXiv:2203.05422 [hep-lat]].

[4] D. Prekrat, "Renormalization footprints in the phase diagram of the Grosse-Wulkenhaar model," Phys. Rev. D **104**, no.11, 114505 (2021) doi:10.1103/PhysRevD.104.114505 [arXiv:2104.00657 [hep-th]].