In this article the authors study quantum features of the “fuzzy sphere.” The term “fuzzy sphere” is known as a matrix realization of a non-commutative sphere in mathematical physics literature, firstly coined by Madore [1]. The fuzzy sphere provides one of the simplest examples of the non-commutative geometry à la Connnes [2]. Note that the “fuzzy sphere” studied in this article does not identify with the conventional definition of either Madore's or Connes'. It represents a non-commutative sphere in “quantum Riemannian geometry,” that is, a recently developed non-commutative geometry by the authors, using quantum groups and the notion of a so-called bimodule connection. For details of their construction of quantum Riemannian geometry, one may refer to the recent textbook [3].

In section 2 of this article, some basic ingredients of their formulation are described. In the following sections the authors calculate “quantum” Levi-Civita connection and propose an action for quantum gravity on the “fuzzy sphere.” For readers who are interested in physical applications of non-commutative geometry this article may provide a novel approach from a foundational perspective.

# References


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