In this article the authors develop the computation of the spectral action on Robertson-Walker metrics [1, 2]. They reveal a novel combinatorial structures in the asymptotic expansion of the spectral action and show that it can be expressed in terms of the so-called Bell polynomials by use of Brownian bridge integrals and the Feynman-Kac formula.

The authors then apply the resultant expression to a multi-fractal arrangement of Robertson-Walker spacetimes over an Apollonian sphere packing. This arrangement is relevant to the so-called "Packed Swiss Cheese Cosmology" [3]; a spectral action for such a model has been reported in [4]. The authors present that the multi-fractal spectral action contains the same terms as in the case of a single Robertson-Walker metric but the coefficients are modified by zeta-regularized terms which are given by values at integers of the zeta function of the fractal string of the radii of the sphere packing; note that the definition of the zeta function is presented in Section 5. In addition to these modifications, the author claim, the multi-fractal spectral action also contains correction terms arising from the poles of this zeta function.

Overall this paper investigates and reveals intriguing interrelations among geometric, analytic, combinatorial and even number theoretic properties of the spectral actions for gravity theories.

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