This extensive lecture note deals with recent studies of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory, and particularly its planar theory, at both weak and strong coupling. Witten's twistor string theory shows that the perturbative $\mathcal{N} = 4$ super Yang-Mills theory can be understood as a topological string theory. This suggests the so-called weak-weak duality between $\mathcal{N} = 4$ super Yang-Mills theory and string theory. On the other hand, Maldacena's AdS/CFT relation shows that the planar $\mathcal{N} = 4$ super Yang-Mills theory at strong coupling corresponds to the weakly-coupled worldsheet theory for superstrings in $AdS_5 \times S^5$. This means a strong-weak duality between $\mathcal{N} = 4$ super Yang-Mills theory and string theory. One of the main goals of this lecture note is to obtain a comprehensive picture of $\mathcal{N} = 4$ Yang-Mills theory in relation with string theory by attempting to understand the gap between the above-mentioned two dualities. It may be desirable for the reader to have knowledge of string theory, however, this note can be useful for those who are interested solely in gauge theories. The note covers a variety of topics in recent developments of the $\mathcal{N} = 4$ planar theory, part of which are made possible by the authors themselves.