Recently it has been well recognized that use of twistor space in an analysis of gauge theories is remarkably efficient and economical in comparison to a conventional method with Feynman diagrams. This is mainly because of Witten's work on the so-called weak-weak duality between topological string theory and $\mathcal{N} = 4$ super Yang-Mills theory; more precisely, between the topological B model on supertwistor space $\mathbf{CP}^{3|4}$ and the perturbative $\mathcal{N} = 4$ super Yang-Mills theory [1]. Since string theory contains gravity, it is natural to seek for a gravitational analog of this duality in the same framework. Due to conformal properties of twistor space, however, a relevant theory naturally becomes conformal supergravity. This was first investigated in [2]. Although conformal supergravity has interesting mathematical properties, the theory is known to be unphysical, *i.e.*, it lacks unitarity to ruin probability interpretation of the quantum theory. A way of breaking the conformal symmetry has been introduced in a formulation called new twistor string theories [3]. In this formulation, the conformal breaking is carried out by use of a particular variable known as an 'infinity twistor.' Extraction of Einstein supergravity is proposed in new twistor string theories but further studies suggest that they are chiral supergravity rather than Einstein gravity [4].

It is along these lines of developments that this article by Mason and Wolf should be understood. In this article, action functionals for self-dual Einstein supergravity is given in terms of supertwistor variables. A meticulous review of self-dual supergravity in twistor space is presented before the construction of actions with $\mathcal{N} = 0, 1, \cdots, 8$ supersymmetries and with (or without) a cosmological constant. The actions are analogous to those of topological B model on $\mathbf{CP}^{3|4}$ with $\mathcal{N} = 4$ and, hence, these can be interpreted as actions for holomorphic Chern-Simons theories, with deformations of twistor space. The deformations can be carried out by use of an infinity twistor; in the article, it is discussed that Einstein gravity must allow a non-degenerate holomorphic contact structure on twistor space and that this structure can directly be defined by the infinity twistor. The formulation of self-dual Einstein supergravity is essentially given in a space of $\mathbf{CP}^{3|8} \times \mathbf{CP}^3$ which is not Calabi-Yau (super)space. Thus it is not clear whether there is a connection between the proposed formulation and string (or twistor string) theories. In the article, other topics, including the gauge group of the self-dual supergravity and a covariant approach of the formulation, are also analyzed.

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