Many years ago Witten showed that the Kaluza-Klein type universe is quantum theoretically unstable (without the existence of fermions), and introduced the so-called "bubble of nothing" spacetime [1]. Given that the universe would be described by a Kaluza-Klein universe, it is still an intriguing problem to understand field theoretic properties in Witten's spacetime and a related subspace, a Lorentzian version of the Hawking wormhole [2]. Along these lines of motivations, in this article the author investigates analytic solutions of the Klein-Gordon equation (or the wave equation for massive scalar fields) on these spaces. Physical studies on these particular fields were carried out long time ago [3] but in this article the same subject is studied in a more mathematically rigorous fashion. The author obtains an explicit form of the solutions in terms of special functions, taking advantage of the global hyperbolicity of the spaces. Physical meanings of the resultant solutions are considered for both massless and massive scalars. Based on these results, the author also proposes scattering operators on these spacetimes. The results are presumably useful for future studies on quantum cosmology.

References

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