In this article the authors show modularity properties in instanton solutions of the so-called Bianchi type-IX cosmological model. Following the previous study [1] and results in [2], the authors investigate the properties of the Bianchi IX gravitational instantons in terms of theta functions. To be more concrete, they derive the so-called Seeley-deWitt coefficients of the heat karnel expansion of the Dirac-Laplacian on the Bianchi IX gravitational instantons and show that these coefficients can be understood as vector-valued modular forms which are, as discussed in detail and with examples, associated to ordinary modular forms. Definitions of the Seeley-deWitte coefficients, the heat kernel of the Dirac-Laplacian, the vector-valued modular forms, etc. are all provided in the article. It is recommended for readers who are interested in arithmetic structures in gravity theories.

References

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